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RESEARCH MEMORANDUM

THEORETICAL INVESTIGATION OF THE PERFORMANCE OF PROPORTIONAL NAVIGATION GUIDANCE SYSTEMS - EFFECT OF METHOD OF POSITIONING THE RADAR ANTENNA ON THE SPEED OF RESPONSE

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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

RESEARCH MEMORANDUM

THEORETICAL INVESTIGATION OF THE PERFORMANCE OF PROPORTIONAL NAVIGATION GUIDANCE SYSTEMS — EFFECT OF METHOD OF POSITIONING THE RADAR ANTENNA ON THE SPEED OF RESPONSE

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SUMMARY

A linear theoretical analysis has been made of the performance of three proportional navigation guidance systems installed in a given supersonic, variable-incidence, boost-glide, antiaircraft missile at Mach numbers of 2.7 (the nominal design value) and 1.3. These guidance systems, which differ in the method of positioning the radar antenna relative to a coordinate system fixed in space, are compared on a basis of the maximum obtainable speed of response of the missile and guidance-system combination consistent with adequate stability.

It is shown that, with the antenna stabilized in space, the effect of component lags on the response is small, so that the speed of response can be made to approach closely that of the airframe alone.

Conversely, if the antenna is not stabilized in space, the obtainable speed of response is limited by stability considerations of the missile and guidance-system combination. By including compensating networks, it is possible to obtain performance comparable to that of the system with the antenna stabilized. However, the response is relatively sensitive to small variations in network time constants and to missile flight-speed variations. Therefore, unless care is taken in selecting gearings and network time constants, instability is likely to occur due to the missile speed decrease during the glide phase.

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INTRODUCTION

It is generally recognized that guiding an antiaircraft homing missile along a proportional navigation trajectory offers certain important advantages, the principal one being the low missile acceleration required during the terminal portion of the trajectory (see, for instance, references 1 and 2). A proportional navigation guidance system produces a missile rate of turn proportional to the rate of rotation in space of the line of sight between the missile and the target. The guidance system is composed of a seeker, which measures the rate of rotation of the line of sight, and a control system, which produces a missile rate of turn proportional to the seeker output. Most systems use a movable radar antenna in the seeker to track the target and measure the rotation of the line of sight in space.

Various methods of positioning and stabilizing the antenna in space have been proposed and numerous isolated system studies have been made of these methods in conjunction with specific missile airframes and control systems. However, it was considered desirable to investigate on a common basis several types of proportional navigation guidance systems in combination with a given supersonic missile configuration. An investigation of this type should permit a more direct comparison of the system characteristics than do the isolated system studies and should lead to a better understanding of the relative importance and interrelation of the various guidance-system parameters.

For this investigation, three proportional navigation guidance systems were selected which differ in method of positioning and stabilizing the radar antenna in space. In one system the antenna and missile rotations are directly coupled and the antenna is not stabilized in space. In the second system the antenna and missile rotations are coupled, but an attempt is made to space stabilize the antenna by rotating it with respect to the missile an equal and opposite amount to the missile yawing motion. In the third system no coupling between the antenna and missile is present, since the antenna is stabilized in space by mounting it with a free gyro. The systems were investigated in combination with a cruciform, variable-incidence-wing missile configuration similar to one for which extensive measured and estimated aerodynamic data are available.

In the selection of a guidance system, a wide variety of characteristics must be considered, such as the size, weight, and reliability of the various components; the effects of nonlinearities and spurious inputs, such as radar noise; and the ability to correct quickly for launching errors and follow closely target maneuvers. The present investigation has been confined to this latter ability. Linearized kinematic studies have indicated that the miss distance due to launching errors and target maneuvers is a direct function of the over-all system time lag (reference 2).

With zero lag no miss distance occurred. Thus, it appeared reasonable to judge the system performance on the basis of the system time lag or speed of response. Accordingly, the missile transient response to a step in the rate of rotation of the line of sight was examined by means of an electronic analog computer. The speed of response and stability are, in a qualitative sense, immediately apparent from the transient response. The optimum responses were determined by varying the parameters of simplified versions of the three guidance systems for the missile at a nominal homing Mach number of 2.7.

Since most air-to-air antiaircraft missiles are of the boost-glide type with guidance in the glide phase only, some consideration must be given to the change in system characteristics with decreasing flight speed. The relative sensitivity of the systems to changes in flight speed therefore was determined by examining the transient response at a Mach number of 1.3 with the optimized guidance parameters as determined at the higher Mach number. The above procedure was repeated with shaping networks added to the systems having the poorer responses in an attempt to improve them.

As noted above, this investigation is confined to a study of the maximum obtainable speed of response which, to a large extent, determines the magnitude of the miss distance due to target maneuvers and launching errors. Another factor that can contribute significantly to the total miss distance is radar noise in the form of target glint or scintillation. In general, the effect of noise is very important and should be the subject of further investigation.

NOTATION

c, C transfer-function coefficients

$D, (\dot{\ })$ $\frac{d(\ )}{dt}$

f, F shorthand notation for numerator and denominator, respectively, of transfer function with constant term equal to unity

I_y moment of inertia in pitch or yaw, slug-feet squared

K transfer-function gearing

L lift or side force, pounds

M pitching moment or yawing moment, pound-feet (or Mach number)

m mass, slugs

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N navigation ratio $\left(\frac{\dot{\gamma}}{\dot{\sigma}}\right)_{\text{steady state}}$

p a variable introduced in the Laplace transformation

T transfer-function time constant, seconds

V flight speed, feet per second

v voltage

α angle of attack or sideslip, radians

γ flight-path angle, radians

δ control deflection, radians

ϵ radar-antenna error angle, radians

ζ damping ratio

θ angle of pitch or yaw, radians

λ angular orientation of radar-antenna axis with respect to missile longitudinal axis, radians

σ line of sight angle in space, radians

ω_n undamped natural frequency, radians per second

Subscripts

A antenna servo in systems IA and IB, or antenna gyro-precessing mechanism in system II

f feed-back gyro in missile control system

G rate gyro

I integrator

m missile-control-system-combination transfer function $\left(\frac{\theta}{v_R}\right)$

N network

R radar receiver

S control servo

α $\frac{\partial(\ )}{\partial\alpha}$

$\dot{\alpha}$ $\frac{\partial(\ )}{\partial\dot{\alpha}}$

δ $\frac{\partial(\ )}{\partial\delta}$

θ missile transfer function $\left(\frac{\theta}{\delta}\right)$

$\dot{\theta}$ $\frac{\partial(\ )}{\partial\dot{\theta}}$

γ missile transfer function $\left(\frac{\gamma}{\theta}\right)$

DESCRIPTION OF GUIDANCE AND CONTROL SYSTEMS

Guidance Systems

A proportional navigation trajectory is obtained when

$$\dot{\gamma}/\dot{\sigma} = N \tag{1}$$

The ratio of the missile rate of turn to the rate of rotation in space of the line of sight, N, is a constant greater than unity and is defined as the navigation ratio. It can be appreciated that, to provide proportional navigation, a guidance system must perform two functions: determine the rate of rotation of the line of sight and develop a missile rate of turn proportional to this quantity. It was stated earlier that most proportional navigation guidance systems use a tracking radar with a movable antenna to measure the rate of rotation of the line of sight. The antenna detects ϵ, the angle between the line of sight and the antenna axis, and the radar receiver produces an output voltage, v_R, proportional to this angle

$$v_R = K_R\epsilon \tag{2}$$

Due to the geometric space-angle relationship, the antenna position in space $(\theta + \lambda)$, the line of sight angle σ and the angle ϵ are related by the following equation (fig. 1):

$$\epsilon = \sigma - (\theta + \lambda) \tag{3}$$

If, in an ideal case, the antenna is rotated so that its angular velocity in space $(\dot{\theta} + \dot{\lambda})$ is proportional to the radar-receiver output voltage, v_R, or

$$(\dot{\theta} + \dot{\lambda}) = Kv_R \tag{4}$$

it can be shown from the solution of equations (2), (3), and (4) that

$$\frac{v_R}{\dot{\sigma}} = \frac{1/K}{1 + (1/KK_R)p} \tag{5}$$

Thus the radar-voltage output is proportional to the rate of rotation of the line of sight but with a time lag inversely proportional to the seeker gearing, KK_R. With large values of this gearing, the lag is small and the antenna closely tracks the target.

In practice, it is not possible to control the antenna exactly as indicated by the idealized equation (4). The three methods of antenna positioning dealt with in this report can be represented in principle by equation (4), but because of the time lags in the various components significant differences in the over-all dynamic characteristics may occur. Figure 2 is a block diagram of the guidance systems. The upper portion, containing the radar receiver and missile-control-system combination, is common to all three systems while the lower portions, which indicate the method of positioning the radar antenna in space (i.e., forming $\theta + \lambda$), are drawn separated from the upper portion to emphasize the differences. The following paragraphs describe the operation and characteristics of the guidance systems in supplying the missile control system with a signal proportional to the rate of rotation of the line of sight $\dot{\sigma}$:

<u>System IA</u>.- In this system, the antenna is not stabilized in space. It is positioned with respect to the missile by the antenna servo, which responds primarily at a rate proportional to the rate-gyro output. The antenna position depends on the radar-receiver output in an indirect manner, since the rate-gyro output is proportional to the radar output, but modified by the dynamics of the missile-control-system combination. The antenna motion, then, is directly coupled to the missile body rotation for this system.

The missile-control-system combination, which will be described later, can take many forms. It is only necessary that it produce a rate of turn, $\dot{\gamma}$, proportional to the radar-voltage output, v_R.

The closed-loop transfer function, derived in appendix A, is:

$$\left.\begin{aligned} \frac{\dot{\gamma}}{\dot{\sigma}} &= \frac{F_AF_GF_ff_\gamma}{(F_AF_G - K_AK_G)F_ff_\theta + \frac{1}{(K_mK_R)}pF_AF_GF_m} \\ \text{where} \qquad & \\ K_AK_G &= \frac{N-1}{N} \end{aligned}\right\} \tag{6}$$

The navigation ratio is determined by the value of the feed-back gearing, K_AK_G, while the forward gearing, K_mK_R, effects only the dynamics. For a fixed navigation ratio and specified component time constants, it is only necessary to vary one parameter in determining the optimum response. Note that the block diagrams of appendix A and figure 2 differ in the sign of the feed-back quantity for this system. For navigation ratios greater than 1, K_AK_G must be positive and less than 1 for the system as shown in the appendix, and negative with an absolute value less than 1 for the system as shown in figure 2.

System IB.- In this system, an attempt is made to stabilize the antenna in space. It is positioned with respect to the missile by the integrating antenna servo, which essentially responds at a rate proportional to the radar-receiver output. The antenna position in space $(\theta + \lambda)$, that is, the antenna position with respect to the arbitrary reference axis, is the resultant of both the antenna-servo motion and the missile-body rotation. The rate-gyro output is subtracted from the radar output to form the input to the antenna servo in order to stabilize the antenna in space, that is, to separate the antenna position from the missile-yawing motion. With unity gearings and zero time lags in the antenna servo and rate gyro, this would be accomplished exactly, since the θ feedback would be canceled at the adder at which $(\theta + \lambda)$ is formed. However, in the practical case, some lags do occur in these components so that the antenna cannot be exactly stabilized in space and some degree of coupling does exist between the antenna motion and the missile-body rotation.

The closed-loop transfer function for this complete system is (see appendix A for derivation)

$$\left.\begin{aligned} \frac{\dot{\gamma}}{\dot{\sigma}} &= \frac{F_AF_GF_ff_\gamma}{(F_AF_G - K_AK_G)F_ff_\theta + \dfrac{1}{(K_m/K_A)}(1 + pF_A/K_AK_R)F_mF_G} \\ \text{where} \\ K_AK_G &- \frac{1}{K_m/K_A} = \frac{N-1}{N} \end{aligned}\right\} \quad (7)$$

It can be seen that, with fixed values for the control system and guidance-component time constants, the response depends on three groups of gearings: K_m/K_A; K_AK_G, the antenna stabilization gearing; and K_AK_R, the seeker open-loop gearing. For a given navigation ratio N, the gearings K_m/K_A and K_AK_G are interrelated. The gearing K_AK_R is independent of N and affects only the dynamics. In optimizing the response at a fixed navigation ratio, then, only two parameters, K_AK_R and either K_m/K_A or K_AK_G, must be considered.

System II.- In this system, the antenna is stabilized in space by mounting it on a free gyro. The antenna is positioned in space by precessing the gyro at a rate proportional to the radar output voltage and its position is independent of the missile-body rotation. The transfer function for this system is

$$\left.\begin{aligned} \frac{\dot{\gamma}}{\dot{\sigma}} &= \frac{(K_m/K_A)F_A F_f f_\gamma}{(1 + pF_A/K_A K_R)F_m} \\ \text{where} \qquad & \\ K_m/K_A &= N \end{aligned}\right\} \tag{8}$$

Thus, the gearing K_m/K_A is fixed for a given navigation ratio, and it is apparent that a large value of the seeker gearing $K_A K_R$ is desirable to hold the over-all system lag to a minimum.

It can be seen that the characteristics of the three systems described above differ primarily in the degree of coupling between the antenna and the missile-body angular motions. In system II the antenna is completely free of the missile yawing motion except for any friction that might exist in the gimbal pivots, an effect neglected herein. In system IB the degree of coupling depends on the time lags and gearings of the antenna servo and rate gyro, while in system IA the antenna position and missile dynamics are directly coupled.

Transfer Functions of Guidance Components

The assumptions made in expressing the transfer functions of the various components of the guidance systems which appear in figure 2 will now be described. The characteristics of dynamic elements such as those used in these systems can usually be represented to high accuracy by a second-order transfer function, containing a gearing, a natural frequency, and a damping ratio, as in the following equation:

$$\frac{\text{output}}{\text{input}} = \frac{K}{1 + (2\zeta/\omega_n)p + (1/\omega_n^{2})p^{2}} \tag{9}$$

Moreover, the second-order term can often be neglected and the resulting first-order transfer function, containing a gearing and a time constant or lag, will still represent the dynamic characteristics adequately over a frequency range from zero to a value which depends on the component natural frequency and damping ratio. This approximation has been used for most of the components of the systems of this report, since their natural frequencies are much higher than the missile short-period natural frequency. For two of the components, the natural frequency is high enough so that the first-order term is also neglected since it is small in comparison with the first-order terms of the other components.

Radar receiver.- It was assumed that the radar receiver responds instantaneously with an output voltage proportional to the error angle, so that the transfer function becomes

$$\frac{v_R}{\epsilon} = K_R \tag{10}$$

Antenna servo and antenna-gyro processor.- A small, high-performance integrating antenna servo was assumed for systems IA and IB. The corresponding unit in system II is the antenna-gyro precessing mechanism. Identical characteristics were assumed for both, a natural frequency of 10 cycles per second and a damping ratio of 0.7, which results in the following approximate first-order transfer function:

$$\frac{\lambda}{v_\lambda} = \frac{K_A}{p(1 + 0.02p)} \tag{11}$$

Rate gyro.- A natural frequency of 20 cps and a 0.7 damping ratio was assumed, resulting in an equivalent first-order time lag of approximately 0.01. The transfer function is

$$\frac{v_G}{\theta} = \frac{K_G p}{1 + 0.01p} \tag{12}$$

Control Systems

The purpose of the missile control system is to produce a missile rate of turn proportional to the radar output voltage. Although the over-all guidance system response depends to some extent on the type of control system, this report is concerned chiefly with antenna positioning methods, and control systems were not investigated in great detail. The following paragraphs briefly describe the characteristics of the control systems considered. Block diagrams of these systems, including the missile, are shown in figure 3 and the derivation of the transfer functions and a more complete discussion are included in appendix B.

No feedback (fig. 3(a)).- In its simplest (and, from a reliability standpoint, the most desirable) form, the control system is composed solely of a servo linked to the missile control surface. The transfer function is

$$\left.\begin{aligned} \frac{\dot{\theta}}{v_R} &= \frac{K_m f_\theta}{F_s F_\theta} \\ \text{where} \qquad K_m &= K_s K_\theta \end{aligned}\right\} \tag{13}$$

<u>Rate feedback (fig. 3(b))</u>.- This system employs the usual method of feeding back the missile angular velocity to increase the low natural damping of supersonic missile configurations. The lead network at the rate gyro is necessary to compensate for the control-servo time lag (see appendix B). With the proper choice of open-loop gearing, $K_S K_\theta K_f$, the damping ratio at $M = 2.7$ can be varied from a missile-alone value of 0.054 to a maximum of 0.8 with only a small effect on the natural frequency over most of this range. The transfer function is

$$\frac{\dot{\theta}}{v_R} = \frac{K_m F_f f_\theta}{F_S(F_\theta F_f + K_S K_\theta K_f f_\theta)/(1 + K_S K_\theta K_f)}$$

where

$$K_m = \frac{K_S K_\theta K_f}{K_f(1 + K_S K_\theta K_f)} \tag{14}$$

<u>Displacement feedback (fig. 3(c))</u>.- This system employs a displacement gyro to feed back the missile angular movement. It is necessary to add the integrator to obtain a missile rate of turn proportional to the radar output voltage for this system (see appendix B). The transfer function is

$$\frac{\dot{\theta}}{v_R} = \frac{K_m f_\theta}{F_I(f_\theta + pF_S F_\theta / K_S K_\theta K_f)}$$

where

$$K_m = K_I/K_f \tag{15}$$

Due to the large value of T_θ, an aerodynamic time constant in f_θ (see equation (17)) which occurs in the first-order term of the denominator, this system introduces a large lag into the over-all response. Although it is possible to compensate for this lag, the stability would be very sensitive to changes in the missile flight speed (see appendix B). Further investigation disclosed that reversing the sign of K_f (i.e., for $v_S = v_I + v_f$) had a beneficial effect on the speed of response of only system IA. Results are presented therefore only for this control system in combination with system IA.

Transfer Functions of Missile and Control-System Components

In the following paragraphs the simplifying assumptions made in arriving at the transfer functions of the components of the control systems, including the missile, will be discussed.

Missile.- The missile is a supersonic, variable-incidence wing, cruciform configuration. The wings and tails are of triangular plan form with 60° vertex angles and the tails are interdigitated 45°. This configuration is the same as the variable-incidence configuration studied in reference 3. The aerodynamic characteristics, derived in reference 3 from tests of a similar configuration, were used in the present study. It is assumed that a perfect roll-stabilization system is provided. For a symmetrical configuration such as this the lateral and longitudinal equations of motion are identical, if the effect of gravity is neglected. The common practice of neglecting this effect has been followed in the present study, and the familiar longitudinal equations of motion are used. Neglecting changes in the forward speed, these equations are

$$\left.\begin{aligned} (L_\alpha + mVD)\alpha - mVD\theta &= -L_\delta\delta \\ (-M_\alpha - M_{\dot\alpha}D)\alpha + (-M_{\dot\theta}D + I_yD^2)\theta &= M_\delta\delta \\ \theta - \alpha &= \gamma \end{aligned}\right\} \tag{16}$$

The following transfer functions, which appear in the guidance- and control-system block diagrams, can be derived from the above equations:

$$\left.\begin{aligned}
&\frac{\theta}{\delta} = \frac{K_\theta f_\theta}{pF_\theta} = \frac{K_\theta(1 + T_\theta p)}{p\left[1 + (2\zeta/\omega_n)p + (1/\omega_n^2)p^2\right]} \\
&\frac{\gamma}{\theta} = \frac{f_\gamma}{f_\theta} = \frac{1 + (2\zeta_\gamma/\omega_{n_\gamma})p + (1/\omega_{n_\gamma}^2)p^2}{1 + T_\theta p} \\
&\text{where} \\
&K_\theta = \frac{L_\alpha M_\delta - L_\delta M_\alpha}{-mVM_\alpha - L_\alpha M_{\dot\theta}} \\
&T_\theta = \frac{mVM_\delta - L_\delta M_{\dot\alpha}}{L_\alpha M_\delta - L_\delta M_\alpha} \\
&\zeta = \frac{I_yL_\alpha - mV(M_{\dot\theta} + M_{\dot\alpha})}{2\sqrt{mVI_y(-mVM_\alpha - L_\alpha M_{\dot\theta})}} \\
&\omega_n = \sqrt{\frac{-mVM_\alpha - L_\alpha M_{\dot\theta}}{mVI_y}} \\
&\zeta_\gamma = \frac{-L_\delta(M_{\dot\theta} + M_{\dot\alpha})}{2\sqrt{I_yL_\delta(L_\alpha M_\delta - L_\delta M_\alpha)}} \\
&\omega_{n_\gamma} = \sqrt{\frac{L_\alpha M_\delta - L_\delta M_\alpha}{I_yL_\delta}}
\end{aligned}\right\} \tag{17}$$

Values of these aerodynamic derivatives and missile transfer-function coefficients for an altitude of 50,000 feet and Mach numbers of 1.3 and 2.7 are listed in table I.

Control servo.- The control servo is approximated by a first-order time lag of 0.05 second, which corresponds to a second-order system having a damping ratio of 0.7 and a natural frequency of about 5 cps. The transfer function is

$$\frac{\delta}{v_S} = \frac{K_S}{1 + 0.05p} \tag{18}$$

Rate gyro.- The characteristics of the rate gyro in the rate feed-back control system are assumed identical to those of the rate gyro in the guidance loop. However, the gearing is designated by K_f and a lead network with a lead constant equal to the control servo lag is added. It is assumed that a relatively small lag is introduced by the network and this term as well as the second-order gyro term is neglected. The transfer function, therefore, becomes

$$\frac{v_f}{\theta} = \frac{K_f(1 + 0.05p)p}{1 + 0.01p} \tag{19}$$

Displacement gyro.- It is assumed that the displacement gyro in the displacement feed-back control system introduces no dynamic effects so that its transfer function is

$$\frac{v_f}{\theta} = K_f \tag{20}$$

Integrator.- A 0.01-second lag is assumed for the integrator, so that its transfer function is

$$\frac{v_I}{v_R} = \frac{K_I}{p(1 + 0.01p)} \tag{21}$$

METHOD AND CONDITIONS OF ANALYSIS

Several methods of determining the stability and response characteristics of closed-loop systems are available (reference 4). However, some of these become overly complicated when applied to the example systems which have multiple loops and dynamic elements in the feed-back paths. Hence, transient responses to a step $\dot{\sigma}$ input were obtained directly, using an electronic analog computer.

In order to provide a consistent basis of comparison, the system parameters were adjusted so that, if possible, there was a 30-percent initial overshoot in the $\dot{\gamma}$ output response. For a second-order system this criterion results in a reasonable compromise between the stability and speed of response. For higher-order systems such as those of this investigation, this criterion also applies as a first approximation, if there is one predominant oscillatory mode.

Most of the results of this investigation are in the form of "optimized" transient responses. These were obtained by varying the gearings of the guidance and control systems to determine the most rapid response consistent with the 30-percent overshoot requirement. According to the results of a simplified trajectory analysis (reference 2), the miss distance due to launching errors and target maneuvers depends directly on the system speed of response or time lag.[1] Therefore, a comparison of the speed of response obtainable with the three high-order systems investigated in this report should indicate their relative merit in counteracting launching errors and target maneuvers.

A navigation ratio of three was selected and held constant throughout this investigation. This value was considered a reasonable compromise between higher ratios, which minimize the miss due to launching errors and target maneuvers, and lower ratios which reduce the effects of noise.

The calculations were performed for Mach numbers of 2.7 and 1.3 at an altitude of 50,000 feet. This Mach number range is representative for this type of boost-glide missile, and the high altitude was chosen as being critical in terms of missile maneuvering capabilities. The transient responses presented for a Mach number of 2.7 (considered the nominal design speed) are the optimized responses as described above. The responses at a Mach number of 1.3 (considered an extreme off-design speed) are for the optimized gearings, as determined at the Mach number of 2.7.

The results include the three guidance systems of figure 2 in combination with the control system having no feedback and rate feedback (figs. 3(a) and 3(b)) for the above conditions. The displacement feedback control system is used only with system IA. Results also include the effect of networks on systems IA and IB with the rate feed-back control system for the nominal design speed condition.

[1]In this type of analysis the complete guidance- and control-system dynamics are approximated by a first-order transfer function and the kinematic equations are linearized. The solution, in the form of a nondimensionalized miss distance, indicates that, with other quantities held constant, the miss distance is directly proportional to the system time lag, in the absence of noise. When a noise input is also considered, however, some amount of lag is desirable. The necessary lag, which depends on the noise characteristics, can be provided by a filter or by decreasing the system internal gearings to obtain a more sluggish response.

RESULTS AND DISCUSSION

Effect of Method of Positioning the Radar Antenna on the Speed of Response

Figure 4 presents the optimized responses, as defined in the previous section, for a Mach number of 2.7. Included are the results for the three guidance systems of figure 2 in combination with the appropriate missile-control systems of figure 3. The corresponding values of the optimum gearings are listed in table II.

System IA (fig. 4(a)).- With either rate feedback or no feedback in the control system, the response for this guidance system is very sluggish. A very small high-frequency damped oscillation is excited initially. If the gearing $K_R K_m$ is increased in an attempt to achieve a 30-percent initial overshoot, the high frequency oscillation becomes unstable in both cases. With displacement feedback, the response is greatly improved. Note in table II that for this case K_f is negative (i.e., $v_g = v_I + v_f$ in fig. 3), so that the control system itself is unstable. Results for a conventional displacement feedback (i.e., with $K_f > 0$) are not shown, as it was found that no improvement in the response could be obtained.

System IB (fig. 4(b)).- The optimized response for this system is rather slow, with a rise time of about 0.7 second. If $K_A K_R$ is increased in an attempt to increase the speed of response, the initial high-frequency oscillation becomes unstable. For values of $K_A K_G$ larger than the optimum, the readily discernible longer period oscillation becomes unstable. The higher damping furnished by the rate feed-back control system effects primarily the high-frequency oscillation. However, since this oscillation is of such small amplitude compared to the over-all response, very little change in the speed of response is apparent.

System II (fig. 4(c)).- This system, with a stabilized antenna, has the most rapid response, with a frequency very nearly that of the airframe alone ($\omega \approx 12$rad/sec). The rate feed-back control system increases the damping to some extent, but it is not possible to obtain good damping and still maintain 30-percent overshoot. By increasing the open-loop control-system gearing, the damping can be increased but the overshoot decreases as is shown in the figure. Further investigation of this system has indicated that this is a characteristic of the variable-incidence missile configuration.

The results of this section indicate clearly the limitations on the maximum speed of response obtainable when the antenna is not stabilized in space (systems IA and IB). The range of usable gearings is limited due to stability considerations so that it is not possible to increase the speed of response to a desirable value. With the antenna stabilized in space (system II), the speed of response is limited only by the design characteristics of the airframe alone.

Effect of Mach Number on the Response

Figure 5 shows the responses at a Mach number of 1.3 with the same component gearing as for the $M = 2.7$ responses. The missile-control-system gearing K_m is, of course, different due to the change in the missile gearing K_θ.

System IA (fig. 5(a)).- For this system, the responses with no feed-back and rate feedback in the control system have remained stable but sluggish. The navigation ratio, being independent of K_m, has remained constant. With the displacement feed-back control system, the response has become violently unstable.

System IB (fig. 5(b)).- Without feedback the response has become unstable. With rate feedback, the response is stable but with poorer damping than at the higher Mach number. The navigation ratio, being a function of K_m, has increased due to the increase of K_θ.

System II (fig. 5(c)).- For this system, the response has remained stable with both control systems. The navigation ratio has increased due to the increase in K_m, and the frequency has decreased due to the decreased missile natural frequency.

The results of the previous two sections illustrate the desirability of stabilizing the antenna in space. A more rapid response can be obtained, and the effects of the decrease in the missile flight speed during the glide phase are not serious. Also, it should be pointed out that the character of the response for this system is independent of the navigation ratio. For the other two systems, the navigation ratio occurs as a factor in the coefficients of the denominator of the transfer function

Effect of Networks

It was shown in the previous sections that a rapid response could not be achieved with the simplified systems in which the antenna is not stabilized in space (systems IA and IB). In this section, the effect of compensating networks to increase the speed of response of the systems incorporating rate feedback in the control system is investigated.

System IA.- It is not immediately apparent what network is necessary to improve the response of this system. However, if it is assumed that the desired response is that of the missile-control-system combination alone such as occurred in the case of system II, then the desired transfer function becomes

$$\left(\frac{\dot{\gamma}}{\dot{\sigma}}\right)_{desired} = \frac{NFff_{\gamma}}{F_m} \tag{22}$$

If this equation is substituted for $\dot{\gamma}/\dot{\sigma}$ in the system transfer function (equations (A6) and (A7) which include the network in the feed-back path), the network necessary to attain the desired response can be determined by solution of the resulting equation. The solution is

$$\frac{f_N}{F_N} = F_A F_G \left[\frac{\frac{N}{N-1}F_f f_{\theta} - \frac{1}{N-1}F_m + \frac{1}{N-1}\frac{pF_m}{K_m K_R}}{F_f f_{\theta}}\right] \tag{23}$$

Retaining only first-order terms and neglecting quantities of small magnitude in the brackets, and introducing small lags for the two lead factors outside the brackets, the necessary network becomes approximately

$$\begin{aligned}\frac{f_N}{F_N} &= \frac{1 + T_A p}{1 + (T_A/10)p}\,\frac{1 + T_G p}{1 + (T_G/10)p}\left[\frac{1 + (3/2)T_{\theta}p}{1 + T_{\theta}p}\right] \\ &= \frac{1 + 0.02p}{1 + 0.002p}\,\frac{1 + 0.01p}{1 + 0.001p}\left[\frac{1 + 1.269p}{1 + 0.846p}\right]\end{aligned} \tag{24}$$

With this network included, the response was optimized by varying $K_m K_R$ and the largest lead constant. The speed of response was increased considerably but still was not as good as for system II. Of the terms neglected in equation (23), the control servo lag is the largest. To improve the response further, an additional lead network was added between the radar receiver and missile control system to compensate for the control servo lag. This network is

$$\frac{v_R'}{v_R} = \frac{1 + T_S p}{1 + (T_S/10)p} \doteq \frac{1 + 0.05p}{1 + 0.005p} \tag{25}$$

The improved response for this system with a lead constant of 1.30 is shown in figure 6(a). However, the system is unstable at a Mach number of 1.3 and it is necessary to reduce the lead constant to 1.272 to maintain stability throughout the Mach number range. The response with the reduced lead is also shown in figure 6(a).

System IB.- It was pointed out earlier that the antenna would be stabilized in space for this system if there were no lags in the rate gyro and antenna servo, and if the product of the gearings of these two components were unity. For these conditions the response is nearly identical to that of system II, as can be seen by inspection of the transfer functions. The system IB transfer function, equation (7) in a slightly altered form, is

$$\frac{\dot{\gamma}}{\dot{\sigma}} = \frac{(K_m/K_A)F_AF_GF_fF_Nf_\gamma}{(K_m/K_A)(F_AF_GF_N - K_AK_Gf_N)F_ff_\theta + (1 + pF_A/K_AK_R)F_mF_GF_N}$$

With no network present,

$$f_N = F_N = 1$$

and, with the conditions stated above,

$$F_A = F_G = K_AK_G = 1$$

so that the transfer function becomes

$$\frac{\dot{\gamma}}{\dot{\sigma}} = \frac{(K_m/K_A)F_ff_\gamma}{(1 + p/K_AK_R)F_m} \tag{26}$$

Except for F_A, this is identical to the system II transfer function (equation (8))

$$\frac{\dot{\gamma}}{\dot{\sigma}} = \frac{(K_m/K_A)F_AF_ff_\gamma}{(1 + pF_A/K_AK_R)F_m}$$

This similarity occurs because, with the assumption of no lags and unity gearings, the first group of terms in the denominator of the system IB transfer function has been eliminated. A similar result can be obtained if, with $K_AK_G = 1$, the network were to introduce leads equal to the lags of the antenna servo and rate gyro, that is,

$$\frac{f_N}{F_N} = F_AF_G \tag{27}$$

The system IB transfer function now becomes identical to that of system II. Since it is impossible to obtain pure leads as required by equation (27), the following lead networks with lags of one-tenth the lead were used:

$$\frac{f_N}{F_N} = \frac{v_N}{v_G} = \frac{1 + 0.011p}{1 + 0.001p}\,\frac{1 + 0.022p}{1 + 0.002p} \tag{28}$$

The lead has been increased slightly to compensate for both the antenna-servo and rate-gyro lag plus the lag introduced by the networks. With these networks, the transient response (fig. 6(b)) becomes almost identical to that of system II, as anticipated. A small decrease in the lead terms improves the rather poor damping with only a small decrease in the speed of response, as also shown in figure 6(b). As was the case for system II, the responses at a Mach number of 1.3 are stable.

From these results, it is apparent that a rapid response can be achieved with all three of the guidance systems, provided that the necessary networks are included in the systems in which coupling occurs between the missile and antenna motions. However, the network lead constants must be carefully adjusted as small variations of these parameters cause large variations in the response. It is generally recognized that the probability of failure of a guidance system is to a large extent dependent on the degree of complexity of the system and the dependability of the various components. In selecting the optimum system consideration must be given to the relative complexity, dependability, and accuracy of the networks necessary for systems IA and IB, and of the gyro-precessing mechanism of system II.

CONCLUSIONS

A linear, theoretical analysis has been made of the performance of three proportional navigation guidance systems installed in a given supersonic, variable-incidence, boost-glide, antiaircraft missile at Mach numbers of 2.7 (the nominal design value) and 1.3. A comparison of the optimized responses of these guidance systems, which differ principally in the method of positioning the radar antenna in space, has led to the following conclusions with regard to the maximum obtainable speed of response (which is the optimum response in the absence of noise) consistent with adequate system stability:

1. With the antenna stabilized in space, the effect of component lags on the system response is small, so that the speed of response can be made to approach closely that of the airframe alone.

2. If the antenna is not stabilized in space, it is necessary to include compensating networks to obtain a speed of response comparable to that with the antenna stabilized.

3. The response of systems in which the antenna is not stabilized in space is relatively sensitive to small variations in network time constants and to missile-flight-speed changes. Unless care is taken in selecting gearings and network time constants, instability is likely to occur due to the missile-speed decrease during the glide phase.

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APPENDIX A

GUIDANCE-SYSTEM TRANSFER FUNCTIONS

In the following section, the frequency-dependent portion of the transfer function will be denoted by f_i/F_i where both the numerator, f_i, and the denominator, F_i, are of the form $(1 + C_1p + C_2p^2 + \ldots + C_np^n)$ and the subscript i refers to the component of the system being considered. For example, the control-system rate-gyro transfer function is

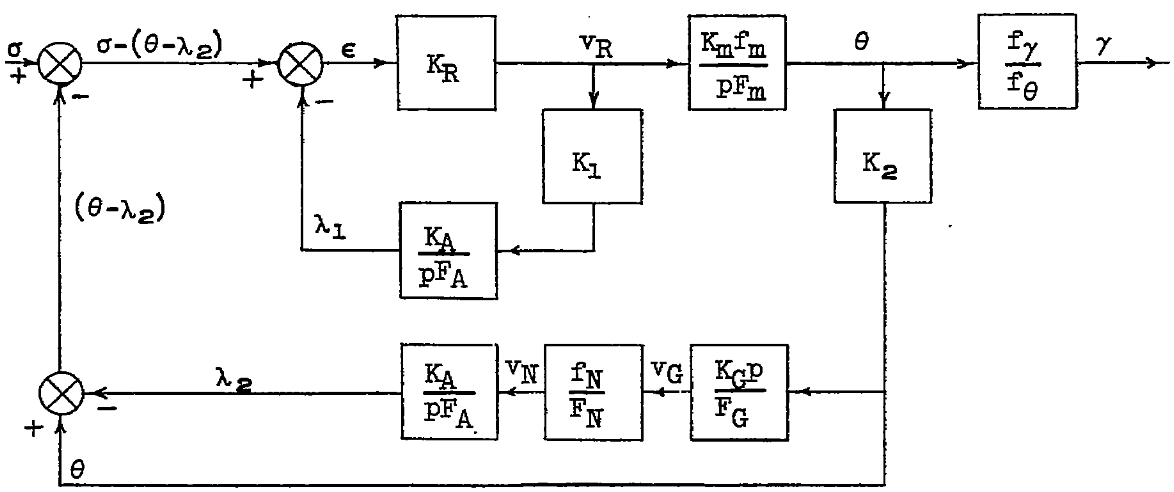
$$\frac{v_f}{\theta} = \frac{K_f(1 + 0.05p)p}{1 + 0.01p} = \frac{K_f f_f p}{F_f} \tag{A1}$$

and the aerodynamic transfer function, θ/δ, is

$$\frac{\theta}{\delta} = \frac{K_\theta(1 + T_\theta p)}{p\left[1 + (2\zeta/\omega_n)p + (1/\omega_n^2)p^2\right]} = \frac{K_\theta f_\theta}{pF_\theta} \tag{A2}$$

This notation is advantageous in determining the closed-loop gearing and the complete-system closed-loop transfer function in a convenient form.

It is also convenient to transform the block diagram of system IB (fig. 2) into the following equivalent form:



The gearings K_1 and K_2 and the network f_N/F_N have been added so that this diagram can represent all the systems considered in this report by the proper choice of values for these parameters. For the first portion of this report where no networks were considered $f_N = F_N = 1$; with $K_1 = K_2 = 1$, the diagram represents system IB; with $K_1 = 0$ and $K_2 = 1$, the diagram represents system IA; and with $K_1 = 1$ and $K_2 = 0$, the diagram represents system II.

Starting from the internal-seeker loop, the closed-loop transfer function can be easily derived, using standard servomechanism methods (reference 5),

$$\frac{v_R}{\sigma - (\theta - \lambda_2)} = \frac{K_R}{1 + K_1K_AK_R/pF_A} = \frac{(1/K_A)F_Ap}{K_1 + pF_A/K_AK_R} \tag{A3}$$

Replacing the seeker loop by its closed-loop transfer function (equation (A3)), the complete system transfer function can be readily determined

$$\frac{\gamma}{\sigma} = \frac{\left(\frac{K_m}{K_A}\right)\frac{pF_Af_m}{p(K_1 + pF_A/K_AK_R)F_m}\frac{f_\gamma}{f_\theta}}{1 + \left(\frac{K_m}{K_A}\right)\frac{pF_Af_m}{p(K_1 + pF_A/K_AK_R)F_m}K_2\left(1 - \frac{K_AK_Gf_N}{F_AF_GF_N}\right)} \tag{A4}$$

Letting $f_m = f_\theta F_f$ ($F_f = 1$ for the displacement feedback and no feed-back control systems, see appendix B), and clearing fractions,

$$\left.\begin{aligned}
\frac{\dot{\gamma}}{\dot{\sigma}} &= \frac{(K_m/K_A)F_AF_GF_fF_Nf_\gamma}{(K_2K_m/K_A)(F_AF_GF_N - K_AK_Gf_N)F_ff_\theta + (K_1 + pF_A/K_AK_R)F_mF_GF_N} \\
&= \frac{\left[\frac{(K_m/K_A)}{(K_2K_m/K_A)(1 - K_AK_G) + K_1}\right]F_AF_GF_fF_Nf_\gamma}{\frac{(K_2K_m/K_A)(F_AF_GF_N - K_AK_Gf_N)F_ff_\theta + (K_1 + pF_A/K_AK_R)F_mF_GF_N}{(K_2K_m/K_A)(1 - K_AK_G) + K_1}} \\
&= \frac{N(1 + c_1p + c_2p^2 + \ldots + c_mp^m)}{(1 + C_1p + C_2p^2 + \ldots + C_np^n)}
\end{aligned}\right\} \tag{A5}$$

where

$$N = \frac{1}{K_2(1 - K_AK_G) + \frac{K_1}{K_m/K_A}}$$

so that

$$\frac{\dot{\gamma}}{\dot{\sigma}} = \frac{NF_AF_GF_fF_Nf_\gamma}{NK_2(F_AF_GF_N - K_AK_Gf_N)F_ff_\theta + \left(\frac{N}{K_m/K_A}\right)(K_1 + pF_A/K_AK_R)F_mF_GF_N} \tag{A6}$$

for system

$$\text{IA:}\quad K_1 = 0,\ K_2 = 1;\ K_AK_G = \left(\frac{N - 1}{N}\right) \tag{A7}$$

$$\text{IB:}\quad K_1 = K_2 = 1;\ K_AK_G - \frac{1}{K_m/K_A} = \left(\frac{N - 1}{N}\right) \tag{A8}$$

$$\text{II:}\quad K_1 = 1,\ K_2 = 0;\ K_m/K_A = N \tag{A9}$$

APPENDIX B

CONTROL-SYSTEM TRANSFER FUNCTIONS

RATE FEED-BACK SYSTEM

Referring to figure 3(b), the closed-loop transfer function is

$$\left.\begin{aligned}\frac{\theta}{v_R} &= \frac{K_S K_\theta f_\theta / p F_S F_\theta}{1 + K_S K_\theta K_f f_\theta f_f / F_S F_\theta F_f}\\ &= \frac{K_S K_\theta F_f f_\theta}{p(F_S F_\theta F_f + K_S K_\theta K_f f_f f_\theta)}\end{aligned}\right\} \tag{B1}$$

With no lead network (i.e., with $f_f = 1$), it was found that only a small increase in the low missile-damping ratio could be obtained with rate feedback due to the rather large control-servo time lag, T_S. However, if f_f is made equal to F_S, this term becomes a common factor of the denominator and the transfer function becomes

$$\frac{\theta}{v_R} = \frac{K_S K_\theta F_f f_\theta}{p F_S(F_\theta F_f + K_S K_\theta K_f f_\theta)} \tag{B2}$$

The stability now depends on only the bracketed quantity which is equivalent to a rate feedback with only the small gyro lag present. With this lead constant present, it is possible to obtain a maximum damping ratio as high as 0.8 at an open-loop gearing, $K_S K_\theta K_f$, of about 0.30. Dividing numerator and denominator by the constant term of the denominator gives

$$\left.\begin{aligned}\frac{\dot{\theta}}{v_R} &= \frac{[K_S K_\theta/(1 + K_S K_\theta K_f)] F_f f_\theta}{F_S(F_\theta F_f + K_S K_\theta K_f f_\theta)/(1 + K_S K_\theta K_f)}\\ &= \frac{K_m(1 + c_1 p + c_2 p^2)}{1 + C_1 p + C_2 p^2 + C_3 p^3 + C_4 p^4}\end{aligned}\right\} \tag{B3}$$

DISPLACEMENT FEED-BACK SYSTEM

Referring to figure 3(c), the closed-loop transfer function is

$$\left.\begin{aligned}\frac{\theta}{v_I} &= \frac{(K_S K_\theta f_\theta)/p F_S F_\theta}{1 + (K_S K_\theta K_f f_\theta)/p F_S F_\theta}\\ &= \frac{K_S K_\theta f_\theta}{K_S K_\theta K_f f_\theta + p F_S F_\theta} = \frac{1/K_f \; f_\theta}{f_\theta + p F_S F_\theta / K_S K_\theta K_f}\end{aligned}\right\} \tag{B4}$$

For proportional navigation with the systems considered in this paper, it is necessary for the pitching velocity to be proportional to the voltage input from the radar in the steady state, rather than the pitch angle as occurs in equation (B4). For this reason, the integrator was introduced and the transfer function becomes

$$\left.\begin{aligned}\frac{\dot{\theta}}{v_R} &= \frac{(K_I/K_f)f_\theta}{F_I(f_\theta + pF_S F_\theta/K_S K_\theta K_f)} \\ &= \frac{K_m(1 + c_1 p)}{1 + C_1 p + C_2 p^2 + C_3 p^3 + C_4 p^4}\end{aligned}\right\} \qquad (B5)$$

This system has a slow response due to the large value of T_θ, which occurs in the first-order term of the denominator. A satisfactory, fast response can be obtained by adding an inner rate feedback, or by introducing a gyro lag equivalent to T_θ which cancels f_θ from the denominator. However, it is difficult to maintain a satisfactory response throughout the speed range without a variable gain and gyro time constant due to the large variation of T_θ with Mach number. For system IA, it was found that a marked improvement in the speed of response was possible at $M = 2.7$ if the gyro gearing was negative. However, due to the abovementioned variation of T_θ with Mach number, the response became unstable at a Mach number of 1.3.

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TABLE I.- MASS AND AERODYNAMIC PARAMETERS OF THE MISSILE
AT AN ALTITUDE OF 50,000 FEET

Parameters	M = 2.7	M = 1.3
L_α, lb/rad	10,300	3,831
L_δ, lb/rad	4,270	1,795
M_α, ft-lb/rad	-6,800	-2,314
$M_{\dot{\alpha}}$, ft-lb-sec/rad	-6.3	-13.6
$M_{\dot{\theta}}$, ft-lb-sec/rad	-26.4	-27.6
M_δ, ft-lb/rad	2,760	2,030
m, slug	6.67	6.67
I_y, slug-ft^2	41	41
V, ft/sec	2,620	1,262
K_θ, 1/sec	0.479	0.614
T_θ, sec	0.846	1.422
ζ	0.0536	0.0972
ω_n, rad/sec	12.9	7.54
ζ_γ	0.0233	0.0322
$\omega_{n\gamma}$, rad/sec	18.1	12.7

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TABLE II.- SYSTEM GEARINGS FOR OPTIMUM RESPONSES AT $M = 2.7$

System	No feedback	Rate feedback	Displacement feedback
IA	$K_AK_G = -0.667$ $K_mK_R = 2.16$	$K_AK_G = -0.667$ $K_mK_R = 24$ $K_SK_\theta K_f = 0.30$	$K_AK_G = -0.667$ $K_mK_R = -6.84$ $K_SK_\theta K_f = -0.884$
IB	$K_AK_R = 3.5$ $K_AK_G = 1.121$ $K_m/K_A = 2.2$	$K_AK_R = 12$ $K_AK_G = 1.121$ $K_m/K_A = 2.2$ $K_SK_\theta K_f = 0.30$	- - - - - - - - -
II	$K_AK_R = 20$ $K_m/K_A = 3$	$K_AK_R = 30$ $K_m/K_A = 3$ $K_SK_\theta K_f = 0.02$	- - - - - - - - -
II Small overshoot	- - - - - - -	$K_AK_R = 20$ $K_m/K_A = 3$ $K_SK_\theta K_f = 0.05$	- - - - - - - - -
With Lead Network			
IA	- - - - - - -	$K_AK_G = -0.667$ $K_mK_R = 20$ $K_SK_\theta K_f = 0.05$	- - - - - - - - -
IB	- - - - - - -	$K_AK_R = 30$ $K_AK_G = 1.0$ $K_m/K_A = 3$ $K_SK_\theta K_f = 0.02$	- - - - - - - - -

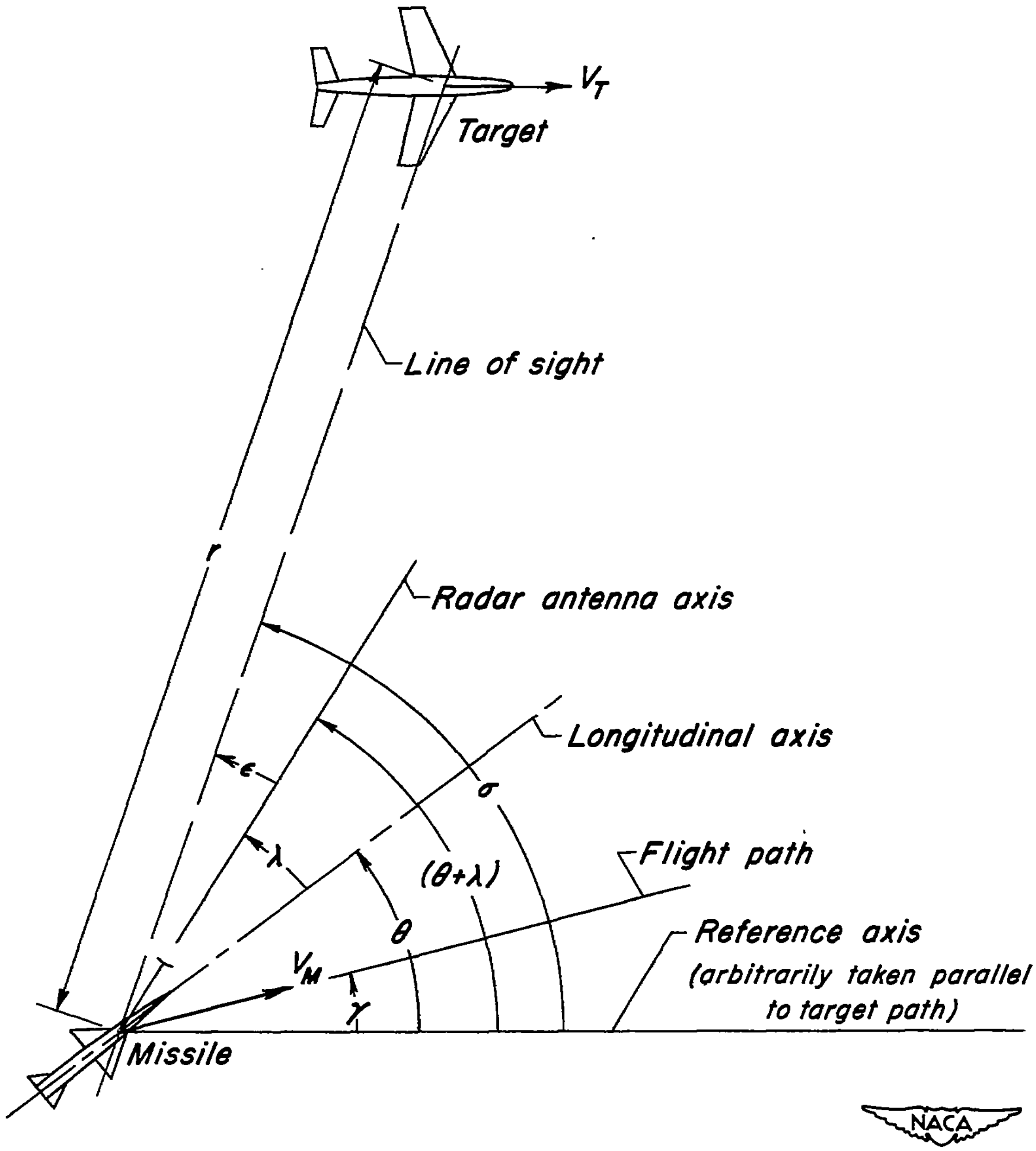


Figure 1.- Geometric variables involved in proportional navigation.

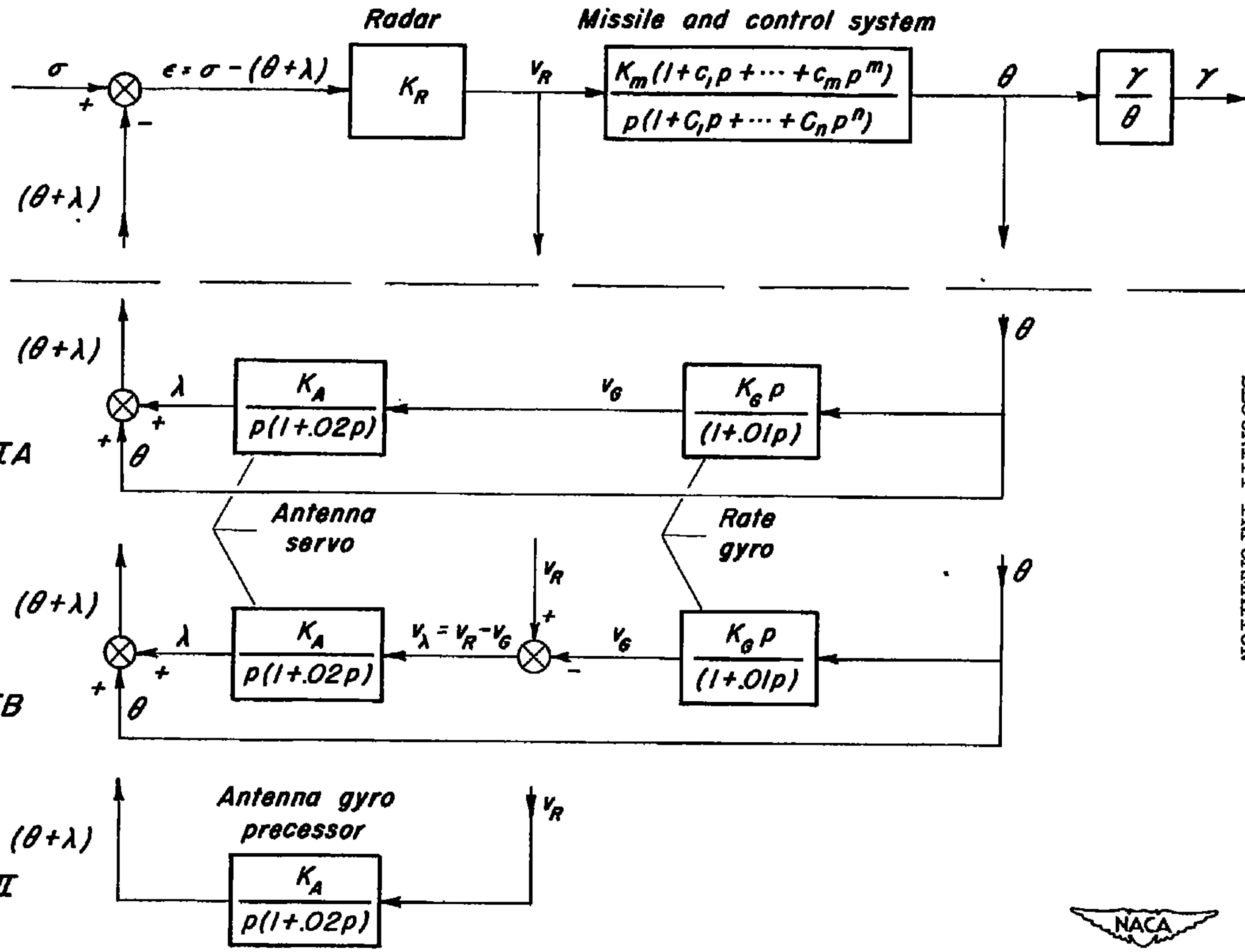
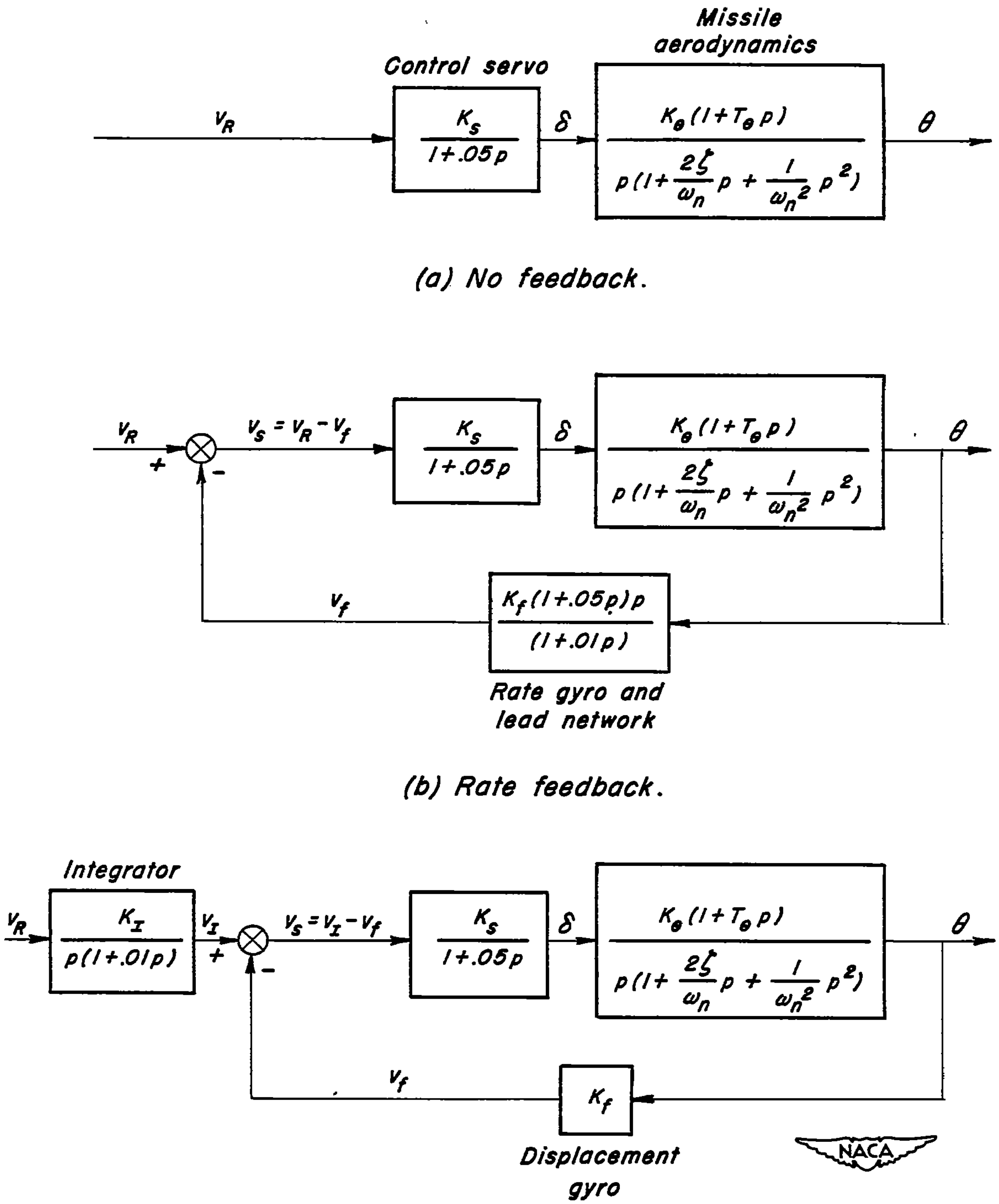
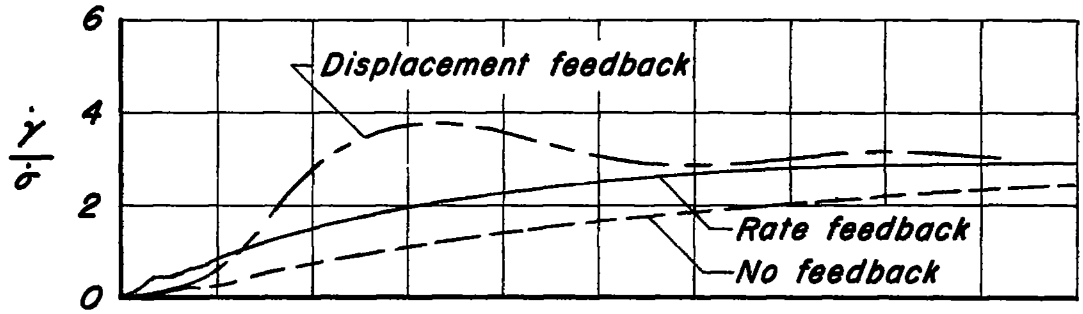


Figure 2.- Block diagrams of proportional navigation guidance systems.

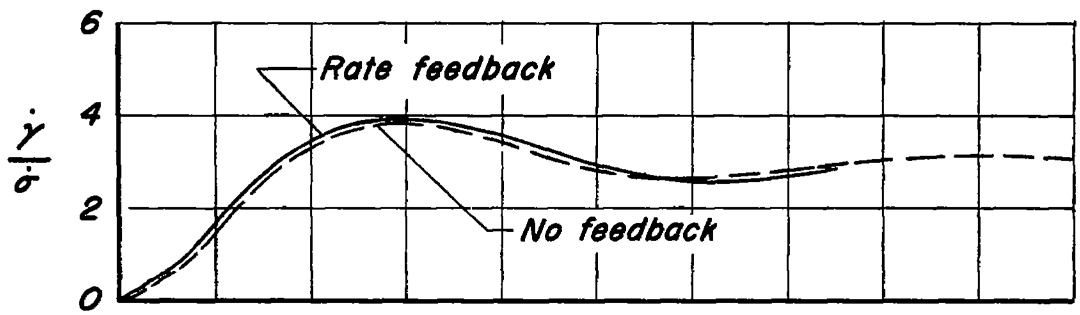


(c) Displacement feedback.

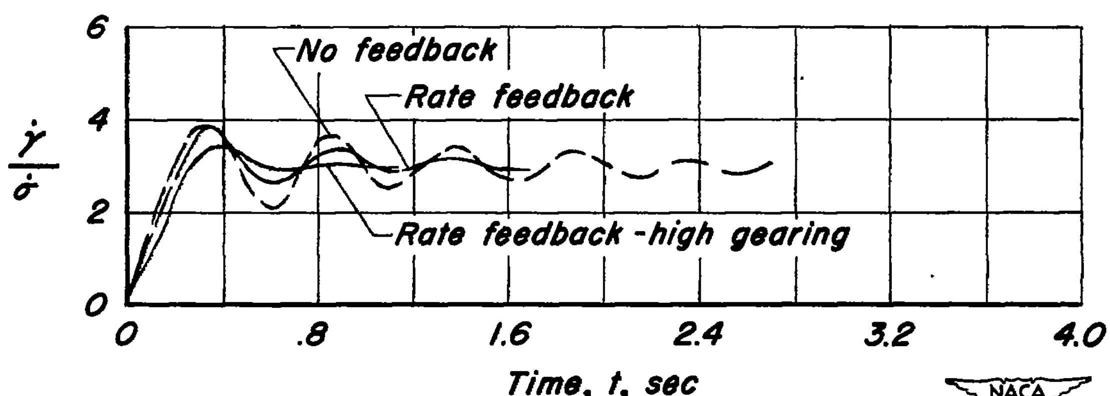
Figure 3.– Missile-control-system combinations.



(a) System IA.

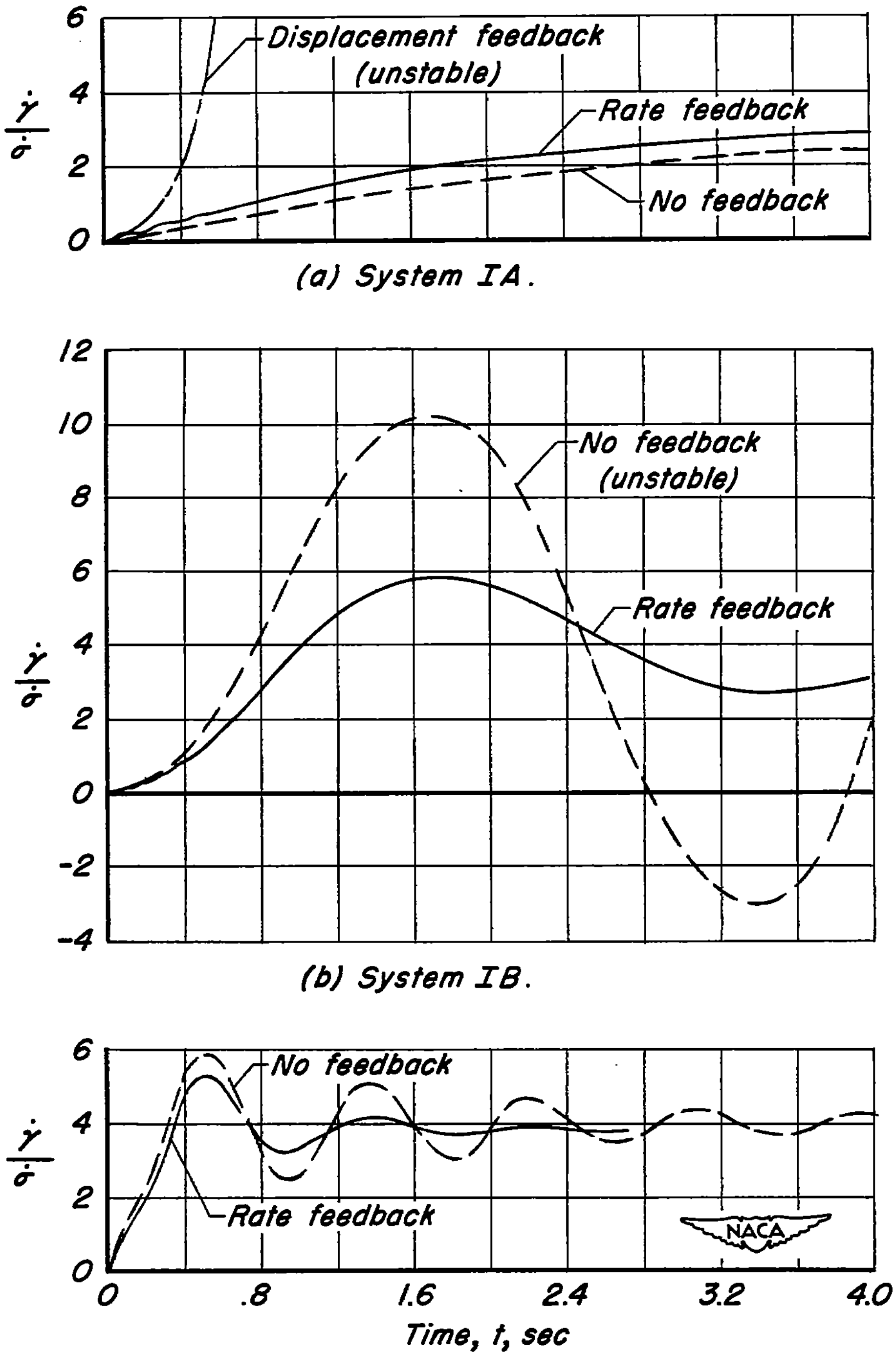


(b) System IB.



(c) System II.

Figure 4.– Transient responses to a step in $\dot{\sigma}$. Mach number, 2.7.

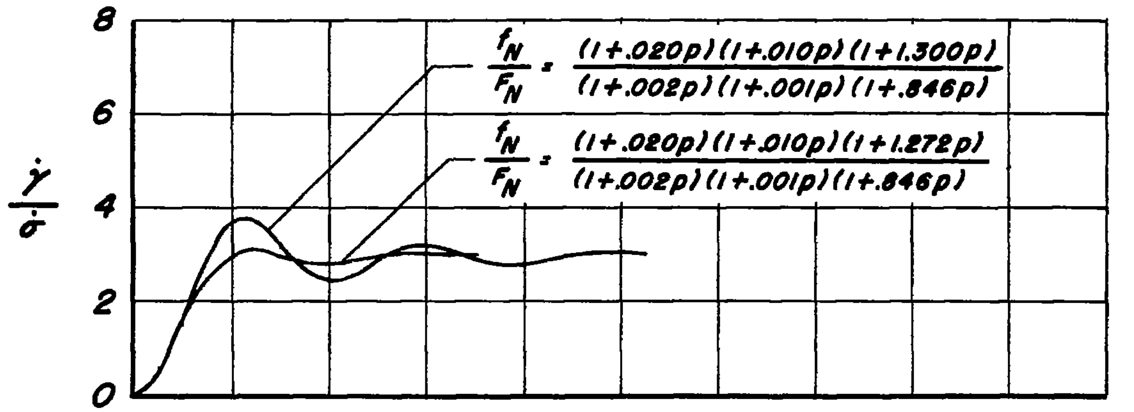


(a) System IA.

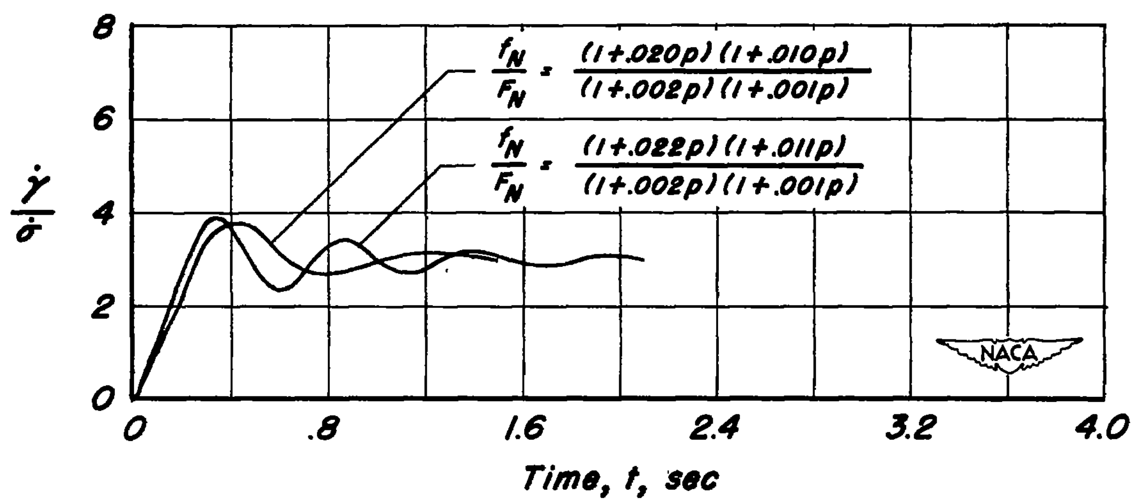
(b) System IB.

(c) System II.

Figure 5.- Transient responses to a step in $\dot{\sigma}$. Mach number, 1.3.



(a) System IA with rate feedback.



(b) System IB with rate feedback.

Figure 6.- Effect of networks on the transient response. Mach number, 2.7.

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